ON THE VANISHING OF EXT AND $\varprojlim^1$

S. BAZZONI

We consider the relations between the vanishing of Ext functors and the derived functors of the inverse limit.

In particular, we focus on the Mittag-Leffler condition on countable inverse limits and we illustrate how this condition can be used to solve a problem on tilting classes of modules and on the class of Baer modules.

Moreover, we discuss how to relate the vanishing of $\varprojlim^n$ with an open problem on Σ-cotorsion modules, namely the problem of determining if pure submodules of Σ-cotorsion modules are cotorsion.

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TILTING THEORY IN ABELIAN CATEGORIES

APOSTOLOS BELIGIANNIS

Abstract. Tilting theory has been established as a fundamental tool in representation theory. Its main aspect is that it gives an effective comparison, via suitable equivalences or dualities at various levels, of usually large parts of the categories which we are interested in. There are several notions of tilting in various settings (mainly for module categories) depending on suitable finiteness conditions. Our aim in this talk is to present a unified treatment of tilting theory in general abelian categories, in some cases without necessarily enough projective and/or injective objects, and to give in this setting the connections with (co)torsion pairs, derived equivalences and, times permits, related homotopical structures.

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Noncommutative Desingularisation of the Generic Determinant

Ragnar-Olaf Buchweitz

In this joint work with Graham Leuschke and Michel van den Bergh we show that the generic determinant admits a noncommutative crepant desingularization by a 'Quiverized Clifford Algebra'.

The talk will explain these terms and show how this result relates to very concrete questions such as the following posed (and mainly answered) by George Bergman: If X is an $n \times n$- matrix with indeterminate entries and $adj(X)$ is its classical adjoint, can one factor $adj(X) = UV$ with noninvertible $n \times n$-matrices U, V ?

Anticyclic operads and Auslander-Reiten translation

Frédéric Chapoton

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We will show that two different constructions lead to the same actions of cyclic groups on some Abelian groups. The first of these constructions lives in the framework of the theory of operads, and more precisely revolves around the notion of anticyclic operad. The other construction is provided by the Coxeter transformation, which is the action induced by the Auslander-Reiten functor on the Grothendieck group of a finite-dimensional algebra.

There are only two examples so far for this relationship. The first one is between the Diassociative operad and the sequence of hereditary algebras of the A_n quivers. This is of course a very classical setting. The other one is between the Dendriform operad and the sequence of incidence algebras of the Tamari lattices. This is related to some more recent developments, such as the theory of cluster algebras.

$\mathbf{G_1T}$-MODULES, AR-COMPONENTS, AND GOOD FILTRATIONS

ROLF FARNSTEINER

Let $(\mathfrak{g}, [p])$ be a finite dimensional restricted Lie algebra, defined over an algebraically closed field k of characteristic $\operatorname{char}(k) = p > 0$. In comparison with representations of complex Lie algebras, one of the main technical problems in the study of $\mathfrak{g}$-modules resides in the lack of information provided by the weight space decomposition

$$M = \bigoplus_{\lambda \in X(\mathfrak{h})} M_\lambda$$

of a $\mathfrak{g}$-module M relative to a Cartan subalgebra $\mathfrak{h} \subset \mathfrak{g}$. While the above provides a $\mathbb{Z}^n$-grading in the classical context, one obtains a gradation by a p-elementary abelian group in the modular setting.

In 1979 J. Jantzen transferred the well-known results by Bernstein-Gel'fand-Gel'fand concerning modules belonging to the category $\mathcal{O}$ of complex semi-simple Lie algebras to Lie algebras $\mathfrak{g} := \operatorname{Lie}(G)$, associated to (smooth) reductive groups G. His main tool was the highest weight category $\operatorname{mod} G_1T$ of G_1T-modules, defined by the first Frobenius kernel G_1 of G and a maximal torus $T \subset G$. Roughly speaking, the objects are finite dimensional $\mathfrak{g}$-modules which admit a grading by a free group (the character group of T) that is compatible with the weight space decomposition relative to the Cartan subalgebra $\mathfrak{t} := \operatorname{Lie}(T)$ of $\mathfrak{g}$.

Using results by Gordon and Green, we show that the Frobenius category $\operatorname{mod} G_1T$ has almost split sequences. Rank varieties are employed to establish the analogue of Webb's Theorem and to investigate components of the stable AR-quiver of $\operatorname{mod} G_1T$ containing modules affording a good filtration.

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Resolutions with structure

Edward L. Green, Virginia Tech, USA

In this talk, I will discuss the interconnection between the structure of projective resolutions of an algebra and the structure of the algebra's Ext-algebra. I will review Koszul and D-Koszul algebras and then discuss Δ-Koszul algebras. I will also mention recent work with E.N. Marcos on $(2-D)$-Koszul algebras.

Koszul algebras and distributive triples

Lutz Hille

Let A be a $\mathbb{Z}$–graded quadratic algebra. Then we can define the quadratic dual algebra $A^!$ and the graded dual B of $A^!$. The Koszul complex $B \otimes A$ (with its canonical differential and a natural grading) is a complex of left projective A–modules. The algebra A is called Koszul, if the Koszul complex is a resolution of the semisimple A–module $A/A_{\geq 1}$. There is a result relating the the Koszul complex to certain triples of vector spaces associated to the algebra A. We formulate this result and derive several consequences. In particular, we obtain a new characterisation of Koszul algebras in terms of distributive triples of vector spaces.

Weighted locally gentle quivers and Cartan matrices

Thorsten Holm
(joint work with C. Bessenrodt; arXiv:math.RT/0511610)

We study weighted locally gentle quivers. This naturally extends gentle quivers and gentle algebras, which have been intensively studied in the representation theory of finite-dimensional algebras, to a wider class of potentially infinite-dimensional algebras. Weights on the arrows of these quivers lead to gradings on the corresponding algebras. For the natural grading by path lengths, any locally gentle algebra is a Koszul algebra.

Our main result is a general combinatorial formula for the determinant of the weighted Cartan matrix of a weighted locally gentle quiver. This determinant is invariant under graded derived equivalences of the corresponding algebras. We show that this weighted Cartan determinant is a rational function which is completely determined by the combinatorics of the quiver, more precisely by the number and the weight of certain oriented cycles. This leads to combinatorial invariants of the graded derived categories of graded locally gentle algebras.

Serre functors, category $\mathcal{O}$, and symmetric algebra

Volodymyr Mazorchuk

This is a joint work with Catharina Stroppel.

Let $\Bbbk$ be a field. A Serre functor on a $\Bbbk$-linear category, $\mathcal{C}$, with finite-dimensional homomorphism spaces is an auto-equivalence, F, of $\mathcal{C}$ which gives isomorphisms

$$\mathrm{Hom}_{\mathcal{C}}(X, FY) \cong \mathrm{Hom}_{\mathcal{C}}(Y, X)^*$$

natural in both X and Y. If A is a finite dimensional algebra, then it is well-known that the bounded derived category $\mathcal{D}^b(A)$ has a Serre functor if and only if gl.dim.$A < \infty$, and if the latter is the case, the Serre functor is just the left derived of the Nakayama functor $A^* \otimes_A \_$. In particular, it follows that there is a Serre functor for all blocks of the BGG category $\mathcal{O}$, associated with a semi-simple complex finite-dimensional Lie algebra. However, since in this case the algebra A is not explicitly given, the Serre functor is not easy to compute. One of our results is the following explicit description of the Serre functor on $\mathcal{O}$ (the geometric counterpart of this result was recently obtained by Beilinson, Bezrukavnkov and Mirkovic):

Theorem. Let T_{w_0} be the global Arkhipov's twisting functor on the regular block of the category $\mathcal{O}$. Then $\mathcal{L}\mathrm{T}_{w_0}^2$ is the Serre functor on this block.

Using the recent results of Khomenko on functors, naturally commuting with translation functors, the above theorem allows us to explicitly describe Serre functors on the regular blocks of the parabolic category $\mathcal{O}$ introduced by Rocha-Caridi. Using the connection between the Serre functors and symmetric algebras one obtains that the endomorphism algebra of the basic projective- injective module in the parabolic block is symmetric. This confirms a conjecture of Khovanov.

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Characterisations of supported algebras.

María Inés Platzeck

We give several equivalent characterisations of left (and hence, by duality, also of right) supported algebras. These characterisations are in terms of properties of the left and the right parts of the module category, or in terms of the classes $\mathcal{L}_0$ and $\mathcal{R}_0$ which consist respectively of the predecessors of the projective modules, and of the successors of the injective modules.

This is a report of joint work with I. Assem, J. A. Cappa, and S. Trepode.

A classification of torsion torsionfree triples in module categories

Manolo Saorín

In 1965 Jans ([2]) introduced the concept of **torsion torsionfree (TTF) triples** in an abelian category. They are triples $(\mathcal{C}, \mathcal{T}, \mathcal{F})$ of full subcategories such that $(\mathcal{C}, \mathcal{T})$ and $(\mathcal{T}, \mathcal{F})$ are both torsion theories. In case the ambient category is the module category $ModA$ over an associative ring A with unit, he gave a bijection between the set of those triples and the set of (two-sided) idempotent ideals of A. A TTF triple as above is called **centrally split** when both constituent torsion theories are split. Jans also proved that the above bijection restricted to another one between centrally split TTF triples of $ModA$ and (ideals generated by) central idempotents of A. On the other hand, the existence of **one-sided split** TTF-triples (i.e. such that only one of $(\mathcal{C}, \mathcal{T})$ and $(\mathcal{T}, \mathcal{F})$ is a split torsion theory) has been known for a long time (cf. [4]). However, the idempotent ideals of A associated to them by Jans' correspondence had not been identified.

In this joint work with Pedro Nicolás (see [3]), we identify those idempotent ideals, thus providing a full classification of one-sided split TTF- triples in module categories. In the particular case when A is an Artin algebra the bijections obtained (for left and right split TTF-triples, respectively) can be obtained one from each other by duality and were considered in an earlier work with Assem ([1]).

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Codimension two singularities of orbit closures for representations of tame quivers

Grzegorz Zwara

Let k be an algebraically closed field, $Q = (Q_0, Q_1, s, e)$ be a quiver and $\mathbf{d} = (d_i) \in \mathbb{N}^{Q_0}$ be a dimension vector. The representations $M = (M_i, M_\alpha)_{i\in Q_0, \alpha\in Q_1}$ of Q with fixed vector spaces $M_i = k^{d_i}$, $i \in Q_0$, form an affine space denoted by $\mathrm{rep}_Q(\mathbf{d})$. The group $\mathrm{GL}(\mathbf{d}) = \prod_{i\in Q_0} \mathrm{GL}_{d_i}(k)$ acts on $\mathrm{rep}_Q(\mathbf{d})$ by

$$(g_i)_{i\in Q_0} \star (M_\alpha)_{\alpha\in Q_1} = (g_{e(\alpha)} \cdot M_\alpha \cdot g_{s(\alpha)}^{-1})_{\alpha\in Q_1}.$$

Then the orbits correspond to isomorphism classes of representations. Let M be a representation in $\mathrm{rep}_Q(\mathbf{d})$ and $\mathcal{X} = \overline{\mathrm{GL}(\mathbf{d}) \star M}$ be the (Zariski) closure of the orbit $\mathrm{GL}(\mathbf{d}) \star M$. The aim of this talk is to present some results concerning codimension two singularities occurring in $\mathcal{X}$, especially if Q is a Dynkin or Euclidean quiver.

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