Acronyme/*short title*	**RUGO**
Titre du projet *(en français)*	Analyse et Calcul des Effets de **Rugo**sités sur les Ecoulements
Titre du projet/*Proposal title* *(en anglais)*	Analysis and Computation of Roughness Effects on Fluid Flows

SOMMAIRE

1. Technical and scientific description of the proposal

1.1 *Rationale*

The RUGO project stems from questions raised by the study of gas and liquids near irregular surfaces. These questions come from various domains of fluid mechanics, from microfluidics to oceanography, and involve various scales. They are described in detail in the next section of this document.

Despite a variety of physical motivations, some typical problems emerge, with relatively well identified mathematical models. These are systems of partial differential equations, for instance of Navier-Stokes type, set in domains whose boundary is "rough". Roughness can be modelled either by a non-smooth surface, or by a smooth but highly oscillating surface. In the latter case, the high oscillations express that the typical lengthscale of the boundary is small compared to the flow one. Once in dimensionless form, this description of roughness involves one or more small parameters.

The main purpose of the project is to build up mathematical and numerical methods for such models. The task is twofold:

At a mathematical level, it means answering non-standard questions on partial differential equations. For instance, when the roughness is modelled by fast oscillations, to understand its mean effect on the flow leads to homogenization problems. In many situations, such problems are nonlinear and nonperiodic, and thus widely open *a priori*. When roughness is expressed through a singular function, one has to find methods that are not too greedy for regularity assumptions. Inspired by recent advances of members of our team, the project contains new ideas to address these issues.

At an applied level, through numerics, we wish to bring more quantitative answers. For instance, in microfluidics, how to express the slip length as a function of the geometrical characteristics of the roughness? Which pattern is the best to minimize the friction? In oceanography, do topography irregularities enhance or decrease Ekman pumping? The underlying numerical codes will take benefit from our homogenization results that allow to derive asymptotic boundary layer systems. They contain no small parameter, hence will be easier to implement.

The RUGO project belongs to the area of applied maths. We want to go from the mathematical theory to the numerical implementation of the models. The composition of the team, to be described later on, reflects this spirit by the complementarity of its seven members. Their knowledge of the domain and recent progress already allows for a detailed and scheduled program, explained further in this document. Fruitful working sessions will start as soon as our internal scientific trips are funded.

1.2 *Background, objective, issues and hypothesis*

The effect of surface irregularities on fluids is ubiquitous in fluid mechanics. It raises questions at both very small scales (tens of nanometers for roughness in a microchannel) and very large scales (tens of kilometers for coastal dynamics). These so-called roughness-induced effects have become a very active field of research in the last ten years, especially with the expansion of microfluidics and biological concerns. Let us detail some of the underlying physical motivations

1.2.1 Physical motivations

<u>a – Microfluidics</u>

The understanding of how rough walls act on fluids is important at least in two ways.

1) *Friction*

The typical diameter of a microchannel is a few micrometers. At such scales, the hydrodynamic resistance gets huge, and any way to reduce friction at the boundary is welcome. In the case of flat surfaces, most of the existing experiments, numerical computations, and theories show that for simple liquids, the so called slip length is of the order of the mean free path [1], [2]. That means a few nanometers and it is therefore neglectible. However, in the case of rough hydrophobic surfaces, the slip length increases significantly, due to trapping of air in the humps of the roughness. An idea shared by many physicists is to take advantage of such surfaces to minimize friction. One should be able to relate the slip length to the main features of the roughness. Some formulas have been proposed in some particular cases [3], but require further justification.

2) *Micromixing*

One of the goals of microfluidics is to obtain a homogenized solute from various chemical products in small quantity. In order to get a good mixing of these products, one idea of current applications is to rely on rough walls. Thanks to the roughness, the flow may go through a stretch-fold process (reminiscent of the famous baker transform), which generates chaos and improves mixing properties. This roughness-induced chaos is of great interest for practical purposes. Even before chaotic regime, roughness has been shown to induce instabilities in laminar microflows, which requires further understanding [4].

b – Geophysics

In the case of oceanography, the “rough walls” are the bottom of the oceans and the coasts, which vary at a scale that is still small compared to the typical extent of an oceanic current. To understand the mean impact of these variations would improve the description of many oceanic features. Let us mention the dynamics of waves or tsunamis, the intensification of western boundary currents, the Gulf Stream separation, or the dynamics of Ekman boundary layers [5]. At the computational level, one would like to filter out the detailed description of the boundary (either the bottom topography or the coasts), and to replace it by a smoothed one with a homogenized boundary condition. Such averaged boundary condition at an artificial boundary is called a *wall law*. The search for wall laws is an active domain of research, not only in oceanography but in fluid mechanics in general.

c – Collision of immersed bodies

The roughness is also involved in the problem of collisions between immersed rigid solids. Indeed, it is known [6] that if the solids have smooth boundaries, and if the incompressible Navier-Stokes equation is used to model the fluid part, no collision should happen, no matter the density of the solids!

This paradoxical result is often explained by physicists through roughness: solids are never perfectly smooth, so that the previous theoretical result falls down. However, no rigorous analysis of this statement has been performed so far.

d – Flows in thin domains

We stress that flows in microchannels belong to the more general flows in thin domains. In this larger setting, roughness is also very important, for instance with regards to lubrication and biological flows.

In lubrication, roughness has an effect on the slip of the mechanical pieces and therefore their weakening. The study of roughness influence has gained an increasing attention from 1960 since it was thought to be an explanation for the unexpected load support in bearings [7]. The coupling of the roughness-induced effects with highly nonlinear phenomena (cavitation, elastic deformation of the surfaces, thermal influence, non-Newtonian behaviour of the bulk fluid...) reveals challenging problems from theoretical, numerical and experimental points of view [8].

For flows in lungs, epithelial cells are sometimes assimilated to roughness, and may change the air dynamics. Moreover the modelling of the respiratory system (which has a great complexity) needs the identification of numerous parameters: in particular, the airway resistance (due to the friction between the gas molecules and the walls of the airways) is a crucial one [9]

1.2.2 Previous mathematical results

The main goal of the RUGO project is to develop mathematical methods and numerical codes to handle the problems described above. We will expand on our strategy in section 1.4. Besides, contributions from members of our team (to be also described in section 1.4), mathematical works have focused so far on wall laws, on the impact of short wavelength bathymetry in ocean models and on thin film flows.

a – Wall laws

Many works are dedicated to wall laws for Stokes or Navier-Stokes equations. In these works, the roughness is modelled by a highly oscillating function, with very small amplitude and wavelength $\boldsymbol{e}$. A homogenized condition is derived as $\boldsymbol{e}$ goes to zero.

A paper of Casado-Diaz, Fernadez-Cara and Simon [10] shows that this homogenized boundary condition is always a Dirichlet boundary condition, under the assumptions that the roughness is “non-degenerate”, and that the velocity field has bounded enstrophy, uniformly in $\boldsymbol{e}$. Therefore, even starting from pure slip at the boundary, one obtains a Dirichlet wall law in the limit.

In the case of Dirichlet boundary conditions, one can push further the analysis, and find the first order approximation, through a boundary layer analysis. It results in a “macroscopic” Navier wall law, at any artificial smooth wall at a distance $O(\boldsymbol{e})$ from the roughness (see [11, 12,13])

We stress that most of these works rely on a stringent periodicity assumption of the roughness. This is of course a huge simplification, both from the point of view of physics and maths. Moreover, the dynamics in the boundary layer is governed by linear (Stokes) equations. This feature is generally shared by flows in thin domains, but not by geophysical fluids, *cf.* the examples section 1.4. In short, one of our ambitions in RUGO is to relax the previous settings, treating random boundaries and nonlinear systems. Moreover, we would like to specify the dependence of the “macroscopic” Navier law on the main features of the “microscopic” roughness.

Of course, the scope of RUGO goes beyond the question of wall laws, *cf.* our workplan.

b – Oceanography

Recent papers are devoted to the effect of non-trivial bathymetry in oceanography. In the context of the quasigeostrophic model, a formal homogenization of small topography variations is performed in [14]. In the context of the water-waves equations, articles [15],[16] deal with highly oscillating bathymetries, respectively in the periodic and random setting. Let us stress that these papers, close to our concerns, are still mostly formal. It is due to the lack of estimates for the water waves equations in norms of low regularity. As one of our ambitions in RUGO is to develop low regularity methods, the project might be of interest in this framework.

c – Thin film flows

Many works are devoted to the treatment of rough boundaries for lubrication theory, from both theoretical and numerical points of view. This area contains a strong interaction between mathematics and engineering sciences. From a theoretical point of view, the derivation and analysis of homogenized models have been performed in a wide range of physical regimes (see for instance [17], [18]): this may includes cavitation, deformation of the surfaces (due to high peak pressures), or the treatment of nano-scales in ultra-thin films with a Boltzmann-type model. From a numerical point of view, Buscaglia and Jai [19] proposed FEM methods for the treatment of the macroscopic flow with corrector adaptations for the microtextures. In the case of cavitation phenomena and elastic deformation, multigrid methods are intensively used for the reduction of the computational costs (see Venner and Lubrecht [20]) but this has a major drawback when taking into account roughness patterns: the computations on the coarse grid cannot use highly oscillating coefficients which do not make sense anymore at this scale, and homogenized coefficients have to be used instead, leading to the homogenization of nonlinear and nonlocal problems. However, all the mentioned mathematical results use a periodic or quasi-periodic framework. Although it may make sense in some industrial situations (due to manufactoring processes), other situations do not fall into this scope (in particular when dealing with ultra-thin film flows) and efforts should be led in this direction. From the industrial point of view, it is as important to note that the majority of fluids used are not Newtonian. The rheology of these flows can then influence the macroscopic behavior. Works of Sac-Epée and Taous [21] and Tichy [22] showed the effect of these complex fluids. It seems thus important to understand their role with respect to roughness issues.

1.2.3 References

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1.3 *Specific aims, highlight of the originality and novelty of the project*

Despite their scattered origins, many questions discussed previously enter the same broad framework.

One should first specify a set of partial differential equations. This may be the incompressible Stokes equations for simple liquid flows in microfluidics. This may be the Navier-Stokes equations coupled with Newton's laws for liquid/solid interaction, or the rotating Navier-Stokes system for geophysical fluids. The description of water waves involves the free surface Euler equations. The study of complex fluids relies on non-Newtonian models, among which Oldroyd type systems. Some physics papers start from bi-fluid models, notably for cavitation issues in lubrication. One can also think of mesoscopic models (typically the Boltzmann equation) for the description of gas dynamics in microchannels, as the Knudsen number is not small.

Once the governing equations have been chosen, one should model the irregular domain in which these equations are set. There are two natural ways to model the irregularities. The first way is to consider a non smooth boundary, say not twice differentiable, or even not Lipschitz. The second way is to model the roughness as a small amplitude/small wavelength perturbation of the boundary. In this framework, a typical example of a rough surface is $\mathbf{z} = \boldsymbol{e}\,\mathbf{R}(\mathbf{x}/\boldsymbol{e},\mathbf{y}/\boldsymbol{e})$ and an asymptotic analysis as $\boldsymbol{e}$ goes to zero is performed. One might say that this point of view is the most commonly used (see the existing mathematical works described in the previous section).

The aim of RUGO is to provide mathematical methods for such problems, and to combine these methods with careful numerical computations to obtain physically relevant information. Of course, this project is not *ex nihilo* and *in abstracto*. It relies on recent progress, especially by the members of this team, and is based on a carefully planned strategy, with short and medium term steps.

Before detailing our work plan, let us stress its broad ambitions and originalities.

1) To develop homogenization techniques for pde's in rough domains.
Indeed, the homogenization problems raised by rough boundaries are not classical, as they distinguish between the direction transverse to the boundary (along which phenomena are localized) and the directions tangential to the boundary (along which the oscillations take place). This anisotropy puts these problems at a crossroad between homogenization techniques and boundary layer techniques. We will take advantage of recent advances in boundary layer analysis as well as on random/nonlinear homogenization. We will go beyond existing works that are mostly limited to a periodic pattern of roughness and a linear setting.

2) To develop low regularity methods adapted to rough domains.
For any modelling of the roughness (non smooth or highly oscillating), one is generally not allowed to differentiate the equations too much. It is therefore necessary to develop methods that are not too greedy for regularity. This is especially urgent for fluid solid interactions: up to now, the existing mathematical theory requires twice differentiable boundaries, and thus forbids the study of roughness effects. Low regularity methods are also needed for the description of boundary layer phenomena,

which usually relies on smooth asymptotic expansions. A combination of several existing tools (elliptic estimates in non smooth domains, weak solutions, defect measures) should yield results in a close future (see next part for all details).

3) To implement numerical codes and derive quantitative information.

The roughness problems are still widely tackled through a direct approach that is very costly due to the short lengthscales involved. One of the ambitions of RUGO is to rely on the homogenization approach to handle such problems. Indeed, homogenization usually leads to a simpler boundary layer problem without small parameter. Such approach has been shown to apply to some academic examples by Achdou, Pironneau and Valentin. Our aim is to handle situations of physical relevance. For instance, we would like to link the homogenized coefficients of wall laws to the main features of the roughness. This is important with regards to microfluidics, for which formulas for the effective slip length have been derived in some special geometries, and require further justification (see references in the previous section). Another application is to quantify the effect of roughness on instability thresholds. Again, we refer to the next section for explicit objectives.

Thus, we believe that the physical understanding of roughness-induced effects is mature enough to allow for a mathematical approach. Such approach will be of both mathematical and physical significance.

1.4 *Scientific program*

Previous paragraphs contained a brief description of the motivations and background of RUGO, as well as its general ambitions, both from the point of view of maths and physics. We present here a list of works to be done in the next three years. This list may of course be refined as underlying difficulties are discovered. Nevertheless, short term and medium term objectives have been clearly identified, so as to stick to the 3 years period of the project.

1.4.1 First task: to derive wall laws for the Stokes and Navier-Stokes system

(Tentative schedule: Years 1-2)

This is a good place to start, as many of the existing studies focus on wall laws. Moreover, the underlying difficulties are mostly linear. It is therefore the natural starting point in the development of homogenization techniques and subsequent numerical codes.

Typically, one considers stationary Navier-Stokes equation in a rough channel, with walls perturbed by a rough profile of characteristic size $e \ll 1$. For instance, in a 2D rough channel, in appropriate coordinates, the boundary is given by $\mathbf{y} = e\,\mathbf{R}(\mathbf{x}/e)$ and one looks for a good homogenized boundary condition at the smoothed boundary $\mathbf{y=0}$.

As mentioned earlier, under the general assumption that the boundary **R** varies in any direction at at least one point, the homogenized boundary condition is at first order of approximation, a Dirichlet boundary condition [1].

When starting from Dirichlet boundary conditions at the rough surface, which correspond to the widespread situation of wetting liquids, it has been shown that such condition can be refined: through a boundary layer analysis, the next order approximation is given by a Navier law, with slip length $\mathbf{O}(e)$ related to an auxiliary boundary layer system without small parameter. Briefly, it is a Stokes type equation, set in a semi-infinite domain ("a humped half plane") corresponding to a dilation near the boundary.

Until recently, the derivation of such Navier law had been performed only for periodic roughness (**R** is periodic), which greatly simplifies the boundary layer analysis, see references section 1.2. In some recent papers by A. Basson and D. Gerard-Varet [2], and D. Gerard-Varet [3], the analysis was extended to random spatially homogeneous boundaries. The main conclusion (that is approximation by a macroscopic Navier law) remains valid, but the papers emphasize strong qualitative differences with the periodic framework. The analysis of the auxiliary system is much more difficult, and the asymptotic behaviour far from the boundary is modified (solutions do not decay exponentially fast in general). The justification of this behaviour involves tools like Saint Venant estimates, the ergodic theorem, or a central limit theorem for weakly correlated random variables. We believe such techniques will be useful for other aspects of this project.

In the line of these works, the following tasks should be achieved if the project is funded:

1) A numerical study of the random setting.

The first thing is to the study the auxiliary boundary layer system. It would be very nice to recover the loss of exponential decay predicted by the theory. Note that as the boundary layer equations hold in an infinite domain, their simulation will require careful methods. The second thing would be to compare the approximations derived from the Dirichlet and Navier wall laws to the true flow. This would extend previous studies, notably [4].

2) A study of quasiperiodic, or almost periodic roughness

Besides the periodic and random stationary ones, the quasiperiodic setting is classical in standard homogenization. It would be of great interest to justify our boundary homogenization in this setting. Preliminary analysis gives hope to prove it in the case of a quasiperiodic pattern satisfying a small divisor assumption. In the more general almost periodic situation, much remains to be understood.

3) A study of non-wetting liquids

One may try to derive the same boundary layer analysis starting from pure or partial slip at the rough boundary, instead of no slip. We already know that for a generic pattern of roughness, the homogenized boundary condition is no slip. But up to our knowledge, the next order correction (involving the boundary layer phenomenon) is still not known, even in the periodic case. In a similar spirit, in relation to rough superhydrophobic surfaces in microfluidics (see section 1.2), one could study the case of a channel with walls having a checkerboard microstructure, that is alternating a Neumann boundary condition (corresponding to slip over the humps of the roughness) and a Dirichlet condition (corresponding to no-slip at the bumps of the roughness), over typical lengthscale ***e***. One may expect as previously a Navier wall law. Physicists are particularly interested on the specific dependence of the slip length on the shape and main characteristics of the roughness (see the formulas suggested in [5] for the checkerboard model). This dependence should be checked numerically, and if possible theoretically. The next task consists in finding the optimal shape that is minimizing friction.

4) Justification of the Hertz model

Still in relation with the homogenization of fluid/solid interaction, we wish to study the case of thin film flows. In the elastohydrodynamic regime, they are modelled by a generalized Reynolds equation, which is modified by a specific nonlocal nonlinearity (Hertz model). Interestingly, this coupled model (which is widely used in industrial applications) is a straightforward adaptation of contact laws in solid mechanics but has never been justified for wet contacts. Moreover, the influence of the roughness patterns on the Hertz model itself is questionable and remains a matter of debate. Thus, we propose to derive in a rigorous way the Hertz model and investigate the dependency of the model with respect to the roughness scales. A numerical validation of the results may be obtained by adapting the Bermudez-Moreno duality algorithm, which was developped by Vazquez and Durany [6].

All computational aspects should be greatly simplified by the experience of L. Chupin and S. Martin which are used to numerical techniques for thin films: see for instance [7], [8] and [9] that deal already with rough boundaries. Moreover, P. Vigneaux has a deep knowledge of microfluidics and developed specific codes for it, see [10].

1.4.2 Second task: to study the role of roughness in the collision of immersed solids

(Tentative schedule: Years 1-2)

This task is based on preliminary discussions between D. Gérard-Varet and M. Hillairet. The experience of the latter one in the field of fluid/solid interaction gives hope for rapid progress in this area. This is a good starting point for the use of low regularity methods, which is a general objective of the RUGO project.

We stress that this task will be pursued in parallel to the one on wall laws. For instance, “theorists” can handle the collision problem while “numerists” address quantitative issues on roughness-induced slip and corresponding friction.

The general topic discussed here is the dynamics of rigid solids immersed in a Newtonian liquid. The fluid part is modelled by the incompressible Navier-Stokes equations, coupled to Newton’s laws for the solid.

At a formal level, it is known since pioneering works of Brenner that such model does not predict any collision between smooth solids. This result is of course paradoxical, as it goes against Archimede’s law. Many physics papers explain this paradox by the fact that the smoothness assumption is never satisfied. Irregularities might therefore explain the collision between solids.

At a mathematical level, the study of fluid/solid interaction is more recent, starting from the pioneering work of Desjardins and Esteban (*cf.* [11, 12]). A justification of the formal no-collision result has been obtained recently by Matthieu Hillairet [13] (see a preliminary result in [14]) for solids whose boundaries are twice differentiable, in 2D configurations.

We stress that the 2D well-posedness for such fluid/solid models also holds in this framework (a notion of weak solutions exists for more irregular solids, but these weak solutions are not known to be unique even before collision). Indeed, a key ingredient of the well-posedness result is the gain of two Sobolev derivatives for the solution of the elliptic Stokes equation. Such result is not true for less than twice differentiable boundaries.

One of the goals of the RUGO project is to analyse the link between roughness and collision within this Navier-Stokes modelling.

1) Well-posedness for rough boundaries.
A first plausible objective is to extend the 2D well-posedness theory to boundaries whose derivative is not differentiable but Holder continuous. Indeed, refined elliptic regularity results hold for such situations, and provide alternative bounds that should ensure existence and uniqueness in an appropriate functional framework.

2) Roughness/Occurrence of collisions.
In a second step, one would like to discuss the collision issue in this Hölderian framework. Again, preliminary formal computations tend to show that there is a threshold in the Hölder exponent of the derivative of the solids boundaries. Above this exponent, no collision can occur. Below this exponent, collisions can occur under the action of gravity. Much remains to be done to go from these formal expectations to a complete mathematical proof. If everything works, it will be the first mathematical justification of roughness effect on collisions.

3) *Boundary condition/Occurrence of collisions.*
In connection with the phenomenon of roughness-induced slip, we wish to re-investigate the collision problem for smooth bodies, but considering a Navier boundary condition at the fluid/solid interface. Previous studies were restricted to a Dirichlet boundary condition, for which a level set formulation of the free boundary can be used. Such approach seems to fail for Navier conditions, raising new mathematical questions.

4) 3D configurations.
Once the 2D analysis is performed, the next objective is to extend it to 3D configurations. This is a long dated task, as the underlying mathematical theory is less understood.

1.4.3 Third task: to understand the connection between roughness and hydrodynamical instabilities.

(Tentative schedule: Years 2-3)

The effect of roughness on hydrodynamical flows has been emphasized notably with regards to stability issues. It is well known from physicists that roughness enhance instabilities, that is decrease instability threshold (this threshold refers to the variation of a parameter, typically the Reynolds number: at low Reynolds, the flow is stable, and above some threshold it gets unstable).

We believe that, after task 1 has been achieved , we will be able to investigate this claim. Indeed, minor modifications of the homogenization techniques on wall laws should allow to go from stationary problems to eigenvalue problems. This should give a way to compute the corrections to the eigenvalues induced by the roughness, and check if it favours instability. We may rely on previous works on eigenvalues of elliptic operators with highly oscillating coefficients [15]. See also [16] on the damping of waves by boundary layers. We may also benefit from the experience of F. Rousset in the theory of hydrodynamical instabilities, and of the numerical skills of P. Vigneaux with regards to applications to micromixing.

We stress that this objective should go with numerical computations so as to valid theoretical results.

1.4.4 Fourth task: to handle geophysical issues and associated nonlinear mathematical problems

(Tentative schedule: Years 2-3)

This task will be pursued once homogenization and low regularity techniques have been further developed through tasks 1 and 2.

Geophysics models often involve small parameters. For instance, solid surfaces have a typical lengthscale that is very small relatively to the typical lengthscale of the flow: let us mention the bathymetry and the shores in oceans, or the surface of the core/mantle boundary inside the Earth. Modelling these surfaces by highly oscillating boundaries is natural, and brings such problems into the frame of our study.

Another source of small parameters: the viscous effects are weaker than the other forces at stake, like the Coriolis force, or the Laplace force in conducting flows inside the Earth.

The main consequence on our roughness studies is that the boundary layer near the rough boundary is usually governed by a nonlinear dynamics. This is in sharp contrast with the aforementioned homogenization problems that are mostly linear. This gives rise to plenty of mathematical and numerical challenges.

In the context of oceanography, the analysis of roughness-induced effects has been performed on various models by D. Gérard-Varet and D. Bresch: the rotating Navier-Stokes equation [17], the quasigeostrophic model [18], the lake equation [19]. The first two papers are similar in spirit to the works on wall laws: the homogenization process involves solutions of an auxiliary boundary layer system. As regards the lake equation, the role of oscillating bathymetry is studied through an additional term in the equations, in a domain without boundary.

The three systems have common features. The oscillations are involved in the leading order description of the solutions (contrary to the case of wall laws). Moreover, they are involved in a nonlinear manner (through a nonlinear boundary layer or cell system). This yields strong compactness difficulties, and requires low order regularity methods. We refer to the use of defect measures in [19], inspired by a previous work of Lions and Masmoudi on Euler equations [20].

So far, we have overcome such difficulties for a periodic pattern of roughness. Our objectives in RUGO are the following:

1) *To extend the previous results to a roughness that is random stationary.*

We will rely on recent advances, and on the strong background of our team on boundary layer and homogenization techniques. AL. Dalibard and F. Rousset have a refined knowledge of homogenization and boundary layer techniques, applied recently to the rotating Navier-Stokes equation [21] [22]. S. Martin has also tackled nonlinear homogenization problems in link with lubrication theory [8].

2) *To do numerical computations on these problems.*

This is of great physical relevance: the dynamics of rotating fluids is a standard topic in fluid dynamics. Our result shows that the roughness modifies nonlinearly the so-called Ekman pumping (a vertical counterflow in the boundary layer). This results in a leading order and nonlinear change in the dissipation of energy. To quantify this change in dissipation is important. A preliminary study on a linearized model (see [23]) has shown that roughness can minimize the energy dissipation. Much remains to be done.

3) Extension to Magnetohydrodynamics (MHD) equations.

Once the previous aspects are clarified, we will consider MHD equations, for which the same techniques should apply, and yield interesting information with respects to the description of the liquid iron in the Earth's core.

1.4.5 Fifth task: to consider more general fluids

(Tentative schedule: Year 2-3)

The tasks we have mentioned earlier focus on incompressible Newtonian models of the Navier-Stokes type. It is of great relevance to apply the same questioning and objectives to other types of models. This does not necessarily require the other tasks to be carried out. The objectives explained below can be for instance pursued in parallel to task 4.

Many flows in thin domains, in which roughness effects are important, involve complex fluids. This is especially the case of biological flows. For instance, blood contains polymers that qualify it as a non-Newtonian flow. Lungs involve both mucus (near the "rough" epithelial cells) and air, inviting to use multiphase models. This gives rise to strong differences, both mathematical (the mathematical theory of complex fluids is recent, thus far from complete), and practical. An example is polymer solutions, which show significant apparent slip in a variety of situations, and can lead to slip-induced instabilities. Another aspect of physical significance that could be added to our work is the impact of surface tension. We also propose to study the influence of roughness over the behaviour of non-Newtonian fluids in the framework of lubrication theory. This could be realized in particular via the homogenization of the Stokes-Oldroyd-B thin film approximation, under the weakest assumptions on the defaults patterns. The conjugate effects of roughnesses and non-Newtonian parameters (in particular the relaxation time) would be investigated in order to understand their influence over the load support (which is the leading imposed parameter in tribology). The skills of L. Chupin, S. Martin and P. Vigneaux in this field will help to define proper models and questioning.

Finally, the study of gas is also of great interest, both at small scale (gas flows in microdevices) and at large scales (spacecraft in rarefied atmosphere), see [24]. Therefore, it is useful to examine the questions described earlier in the context of compressible models, and even Boltzmann models, as the Knudsen number is not small. One of the problems to address within RUGO is to determine effective boundary conditions for such kinetic models in the presence of roughness. It is known since pioneering works of Maxwell (see also the recent paper [25]) that pure specular reflection at the boundary corresponds to pure slip at the macroscopic level, whereas diffuse reflection corresponds to no slip. We will try to determine which boundary condition emerges from the homogenization of roughness. The macroscopic result of Casado-Diaz and coauhtors suggests that it is diffuse reflection, no matter the condition at the rough boundary. Special attention will also be paid to higher order corrections.

1.4.6 References

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1.5 *Expected results and potential impact*

On the basis of the program described in the last section, we expect the following objectives to be achieved:

1) *To derive wall laws for non periodic roughness.*

Random roughness: to confirm numerically the theoretical loss of exponential decay in the boundary layer. To show numerically that the theoretical Navier wall law yields a better approximation than the Dirichlet one.

Quasiperiodic roughness: to justify theoretically and through numerics the Navier wall law.

2) *To understand the effective slip length found at rough hydrophobic surfaces*

To justify a limit Navier wall law for channels with patches of slip and pure slip at the boundary.

To recover numerically (and if possible rigorously) formula for the slip length in terms of the main features of the roughness. Some formulas already exist for typical configurations, and will provide criteria for the success of our homogenization approach.

3) *To relate regularity and collisional properties of immersed solids.*

To provide a theorem of collision/no-collision result depending on boundary regularity.

4) *To justify the connection between roughness and sensitivity of channel flows to instability.*

To provide for some typical flows a theorem showing that roughness decreases the instability threshold.

5) *To obtain models for oceanography that account for the global effect of bathymetry and shores*

To extend theorems on the rotating fluids, the quasigeostrophic model and the lake equation to random irregularity at the boundaries.

To quantify the dissipation induced by the roughness, especially in respect to the classical Ekman friction term.

6) *To derive effective boundary conditions for complex fluids and gas at a rough boundary.*

One of the criteria of success (with respect to gas flows) would be a homogenization theorem for the Boltzmann equation in rough domains.

More details are of course provided by the scientific program of the previous section.